Division **E**

Mathematical Olympiads for Elementary & Middle Schools

January 16, 2018

Contest **3**

Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: _____

3A Add: 531 + 642 + 753 + 864 + 975.

3B The first 40 odd counting numbers are written. How many times does '3' appear as a digit?

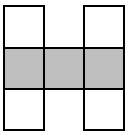
3C Ashley has a rectangle made out of paper that is 8 cm by 12 cm. She folds it in half twice, first vertically and then horizontally. The new rectangle looks just like the original rectangle but smaller. What is the area of the new smaller rectangle in square cm?

Name: _____

Answer Column
3A
3B
3C cm^2
3D
3E
Do Not Write in this Space. *For PICO's Use Only.* *SCORE:*

– Page may be folded along dotted line. –

3D In the figure, the whole numbers from 1 through 7 are to be placed, one per square. The sum of the numbers in the left column, the sum of the numbers in the right column, and the sum of the numbers in each diagonal are the same. What is the least possible product of the numbers across the gray row?



– Page may be folded along dotted line. –

3E In the following cryptarithm, each different letter represents a different digit in the 6-digit numbers. If B ≠ 0, what is the least sum possible?

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  BETTER
  BITTER
  BATTER
+BUTTER
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SOLUTIONS AND ANSWERS

3A **METHOD 1** *Strategy: Write in expanded form and then find the sum.*

531 + 642 + 753 + 864 + 975 =

(500 + 30 + 1) + (600 + 40 + 2) + (700 + 50 + 3) + (800 + 60 + 4) + (900 + 70 + 5) =

(500 + 600 + 700 + 800 + 900) + (30 + 40 + 50 + 60 + 70) + (1 + 2 + 3 + 4 + 5) =

(3500) + (250) + (15) = **3765**.

METHOD 2 *Strategy: Decompose and compose by place value using a table.*

	Hundreds	Tens	Ones
	5	3	1
	6	4	2
	7	5	3
	8	6	4
	9	7	5
Totals	35	25	15

Finally, 3500 + 250 + 15 = 3765.

FOLLOW UP: Four numbers add to 2018. Three of the numbers are 135, 351, and 513. Find the fourth number. [1019]

3B **METHOD 1** *Strategy: Count using place value.*

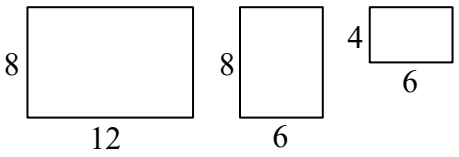
The first 40 odd counting numbers begin with 1 and end with 79. The number '3' will appear in the tens place 5 times (31, 33, 35, 37, and 39) and it will appear in the ones place 8 times (3, 13, 23, 33, 43, 53, 63, and 73). Therefore the number '3' will appear 5 + 8 = **13** times in the first 40 odd counting numbers.

METHOD 2 *Strategy: Make a list and count.*

List the first 40 odd counting numbers: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79. The number of times that the digit '3' appears is 13.

FOLLOW UP: How many of the first 40 even counting numbers contain the digit '4'? [12]

3C **METHOD 1** *Strategy: Draw diagrams to represent the folded paper.*



The area of the final rectangle is 4 × 6 = **24 cm²**.

3A **3765**

3B **13**

3C **24 cm²**

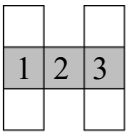
3D **6**

3E **512,024**

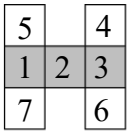
METHOD 2 *Strategy: Use spatial reasoning.*
The area of the original rectangle is $8 \times 12 = 96$ cm^2. When it is folded in half, the area is cut in half and $96/2 = 48$ cm^2. When the paper is again folded in half the area is once again cut in half and $48/2 = 24$ cm^2.

FOLLOW UP: Lisa had a square garden that had an area of 16 square feet. She extends the length of the garden by 2 feet and the width by 3 feet. What is the perimeter of the garden? [26 feet]

3D *Strategy: Use number sense applied to trial and error.*
Since we wish to have the least possible product for the three numbers in the gray row, try using 1, 2, and 3.



The sum of the numbers in the first column must equal the sum of the numbers in the second column. Therefore, the sum of the two remaining numbers in the first column must be 2 more than the sum of the two missing numbers in the second column. The missing numbers in the first column must be 5 and 7 while the two missing numbers in the second column must be 4 and 6.



The diagonals also have the same sum ($5 + 1 + 7 = 13$). One possible arrangement for the four remaining numbers can be seen in the second diagram. The product of the numbers in the gray row is $1 \times 2 \times 3 =$ **6**.

FOLLOW UP: Use the same information as in the original problem with the additional constraint that the sum of the numbers in the gray row equals the other sums. Find the least possible product of the numbers in the gray row. [28]

3E *Strategy: Use place value and number sense to problem solve.*
Since the question asks for the least sum possible, start with the hundred-thousands place and assign B the least possible value, which is 1, since $B \neq 0$. Assign the letters in the ten-thousands place with the least remaining unused digits. Since E repeats in the tens place, assign E = 0. Then assign 2, 3, and 4 to I, A, and U in some order. As a result, the next least available digits for T and R are 5 and 6, respectively. Therefore, the least possible sum is **512,024**.

$$\begin{array}{r} 105506 \\ 125506 \\ 135506 \\ +\ 145506 \\ \hline 512024 \end{array}$$

NOTE: *Other FOLLOW-UP problems related to some of the above can be found in our three contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.*